

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

ated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Project (0704-0188), Washington, DC 20503.

## AD-A240 878



2 REPORT DATE  
August 1991

3 REPORT TYPE AND DATES COVERED  
professional paper

5 FUNDING NUMBERS

In-house funding

NOT ALL FUZZY SET OPERATIONS HAVE WEAK HOMOMORPHIC  
RANDOM SET COUNTERPARTS

6 AUTHOR(S)

I. R. Goodman

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Ocean Systems Center  
San Diego, CA 92152-5000

8 PERFORMING ORGANIZATION  
REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Naval Ocean Systems Center  
San Diego, CA 92152-5000

10. SPONSORING/MONITORING  
AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

This paper is a further contribution to the problem of determining which fuzzy set operations are most appropriate for modeling a given situation.

In review, a choice function, as defined by the author, is a mapping from the class of all fuzzy subsets of a given base space into the class of all random subsets of the same space such that any fuzzy set corresponds to some equivalent random set, where equivalence is with respect to membership function and one point coverage function, respectively. This, in turn, induces an equivalence class relation over the class of all random subsets of the given space. In previous work, a number of characterizations have been obtained for these fuzzy set and ordinary (and hence, random) set operations which have weak (i.e., up to the equivalence described above) homomorphic correspondence, relative to certain families of choice functions and joint distributions of random sets. In summary: 1. It has been demonstrated that all ordinary set operations have natural fuzzy set extensions which lead to weak homomorphic representations. Specific constructions of these operations have been exhibited. 2. Most commonly used fuzzy set operations can be shown to arise as both multivalued logical extensions of and weak homomorphic (or homomorphic-like) images of ordinary set operations. These include: all classes of intersections and unions, and more generally, all classes of Cartesian products and sums, in the forms of triangular norms and conorms, respectively; all combinations of these operations with possibly subtractive complementation, for certain classes of triangular norms and conorms; various relations between fuzzy sets including subsetting, projections, and conditioning and sectioning; and many functional and relational transforms of fuzzy sets.

Thus, it had been conjectured that all fuzzy set operations possess some weak homomorphic counterpart within ordinary set operations. In this paper it is shown that this is not at all the case. By judicious use of the above mentioned characterization theorems, a large class of counter-examples may be constructed. The essential idea here is to begin with some ordinary set operation which leads to many distinct fuzzy set extensions. Then it is shown that, in general, some of these fuzzy set operations, no matter which choice function family is used, cannot be placed in a weak homomorphic relationship to the original ordinary set operation. Specific examples are obtained for the particularly important cases of unary functional composition operations on fuzzy set membership functions and certain simple binary fuzzy set operations.

Published in *Proceedings of NAFIP-1*, May 1982.

14. SUBJECT TERMS

fuzzy set  
algebra

17. SECURITY CLASSIFICATION  
OF REPORT

UNCLASSIFIED

18. SECURITY CLASSIFICATION  
OF THIS PAGE

UNCLASSIFIED

UNCLASSIFIED

15. NUMBER OF PAGES

16. PRICE CODE

20. LIMITATION OF ABSTRACT

SAME AS REPORT

### 91-11477



21a NAME OF RESPONSIBLE INDIVIDUAL

I. R. Goodman

21b TELEPHONE (include Area Code)

(619) 553-4014

21c OFFICE SYMBOL

Code 421

Accession For	
NTIS CRA&I	<input checked="" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By <i>NO SC</i>	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
<i>A-1</i>	<i>21</i>



DOCUMENTLESS INPUT